

CHARACTERISTICS OF AN ELECTRIC-ARC HEATER WITH A
SECTIONED CHANNEL OF VARIABLE RADIUS AND A DISTRIBUTED
GAS FLOW RATE

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Equations are obtained for calculating the electrical and thermal characteristics of a positive column with allowance for the law of variation of the radius of the arc, the physical properties of the gas and its distribution along the channel, the initial distribution of the heat-conduction function, and the current.

The urgency of research on electric-arc heaters (EAH) having sectioned channels of variable radius and distributed gas flow rates was shown in previous papers [1-3]. The present paper is an extension of [1-3] and is devoted to a theoretical and experimental study of the influence of the law of variation of the radius of the arc on the electrical and thermal characteristics of an EAH.

In [1], from a joint solution of the equations of conservation of energy and of continuity and Ohm's law, an equation was found for calculating the electric field strength in an EAH channel:

$$E = I [F_1^2 (4\pi^2 R_0^4 \sigma_s + 2cI^2 \Psi)]^{-0.5}, \quad (1)$$

where

$$F_1 = A_1 \gamma_1 \zeta^2 \alpha^{-\frac{a^2 \mu_1^2}{k_1 l}}; \quad \Psi = \int_0^z \frac{\zeta^2 dz}{\alpha F_1^2};$$

$$\alpha = 1 + k_1 l z; \quad \gamma_1 = \int_0^1 \Phi_1(\mu_1, x) x dx; \quad a^2 = \frac{\pi l}{G_0^* h_s}; \quad \beta = \frac{k_1 l}{a^2};$$

$$c = \frac{\pi R_0^4 \sigma_s l}{G_0^* h_s}; \quad \sigma_s = \frac{\partial \sigma}{\partial S} = \text{const}; \quad h_s = \frac{\partial h}{\partial S} = \text{const}.$$

The dependence of μ_1 on β and an expression for Φ_1 are presented in [1].

We determine E for the case when the law of variation of the radius of the arc can be represented in the form

$$\zeta^2 = \zeta_0^2 (b_0 + b_1 \alpha^m + b_2 \alpha^{2m})^{-1}, \quad b_0 + b_1 + b_2 = 1.$$

Depending on the magnitudes and signs of the coefficients b_0 , b_1 , and b_2 and the exponent m , this equation can describe different cases of the variation of the radius of the arc. Substituting the expression for ζ into (1), we obtain the equation

$$E = \frac{E'_\infty (b_0 + b_1 \alpha^m + b_2 \alpha^{2m})}{\sqrt{2 \sum_{i=0}^2 \frac{b_i \alpha^{im}}{2 + \frac{im k_1 l}{a^2 \mu_1^2}} - \left[2 \sum_{i=0}^2 \frac{b_i}{2 + \frac{im k_1 l}{a^2 \mu_1^2}} - \left(\frac{E'_\infty}{E_0} \right)^2 \right] \alpha^{-\frac{2a^2 \mu_1^2}{k_1 l}}}}, \quad (2)$$

where

$$E_0 = \frac{I}{2\pi R_0^2 \zeta_0^2 \sigma_s A_1 \gamma_1}; \quad E'_\infty = \sqrt{\frac{a^2 \mu_1^2}{\zeta_0^2 c}}.$$

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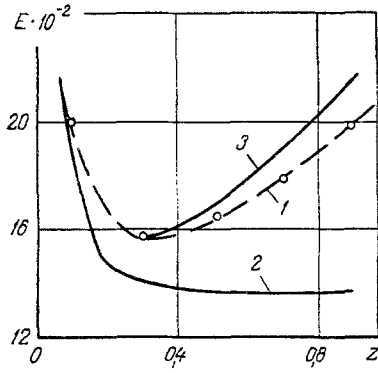


Fig. 1

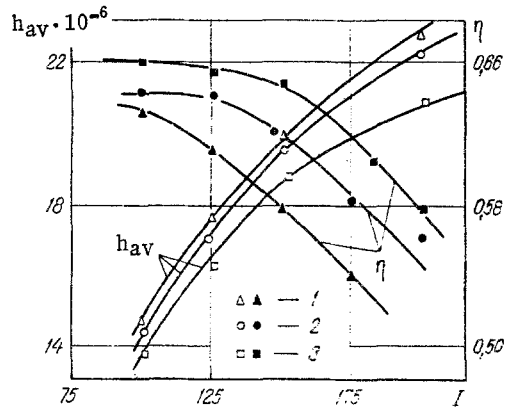


Fig. 2

Fig. 1. Distribution of E along the channel with $I=150$ A, $G_a = 0.8 \cdot 10^{-3}$ kg·sec $^{-1}$, $G_o = 0.2 \cdot 10^{-3}$ kg·sec $^{-1}$, $G_i = 0.12 \cdot 10^{-3}$ kg·sec $^{-1}$, and $k_2 = -6$ m $^{-1}$; 1) experimental curve of [3]; 2) calculation by (7); 3) by (2) for the case of $\zeta^2 = \zeta_0^2(1 + k_1 lz)^{-1}$. E , V·m $^{-1}$.

Fig. 2. Dependences of h_{av} and η on current for different gas flow rates, kg·sec $^{-1}$: 1) $G_a = 1.2 \cdot 10^{-3}$, $G_i = 0.2 \cdot 10^{-3}$, $G_o = 0.2 \cdot 10^{-3}$; 2) $G_a = 1.4 \cdot 10^{-3}$, $G_i = 0.2 \cdot 10^{-3}$, $G_o = 0.4 \cdot 10^{-3}$; 3) $G_a = 1.6 \cdot 10^{-3}$, $G_i = 0.2 \cdot 10^{-3}$, $G_o = 0.6 \cdot 10^{-3}$; h_{av} , J·kg $^{-1}$; I , A.

Using (2), one can find equations for calculating the distribution of the heat-conduction function

$$S(r, z) = \frac{I\Phi_1\left(\mu_1, \frac{r}{\zeta}\right)P}{2\pi R_0^2 \zeta_0^2 \sigma_S \gamma_1 E'_\infty} \quad (3)$$

the average-mass values of S and h

$$S_{av} = S_* + \frac{IP}{\pi R_0^2 \zeta_0^2 \sigma_S E'_\infty} \quad (4)$$

$$h_{av} = h_* + \frac{h_s IP}{\pi R_0^2 \zeta_0^2 \sigma_S E'_\infty} \quad (5)$$

and the heat loss

$$q = \frac{-I\zeta \left[\frac{d\Phi_1\left(\mu_1, \frac{r}{\zeta}\right)}{dr} \right]_{r=\zeta} P}{R_0^2 \zeta_0^2 \sigma_S \gamma_1 E'_\infty} \quad (6)$$

Here

$$P = \sqrt{2 \sum_{i=0}^2 \frac{b_i \alpha^{im}}{2 + \frac{imk_1 l}{a^2 \mu_1^2}} - \left[2 \sum_{i=0}^2 \frac{b_i}{2 + \frac{imk_1 l}{a^2 \mu_1^2}} - \left(\frac{E'_\infty}{E_0}\right)^2 \right] \alpha^{-\frac{2a^2 \mu_1^2}{k_1 l}}}$$

Equations (2)-(6) allow one to calculate the electrical and thermal characteristics of the positive column and their dependence on the physical properties and flow rate of the gas, the current, the channel geometry, the initial distribution of S , and the law of variation of the radius of the arc along its length. With $b_1 = b_2 = 0$, $m = 0$, and $b_0 = 1$ we get from (2) the well-known law of distribution of the electric field strength in a cylindrical arc with distributed gas injection [4]:

$$E = \frac{E'_\infty}{\sqrt{1 - \left[1 - \left(\frac{E'_\infty}{E_0}\right)^2 \right] \alpha^{-\frac{2a^2 \mu_1^2}{k_1 l}}} \quad (7)$$

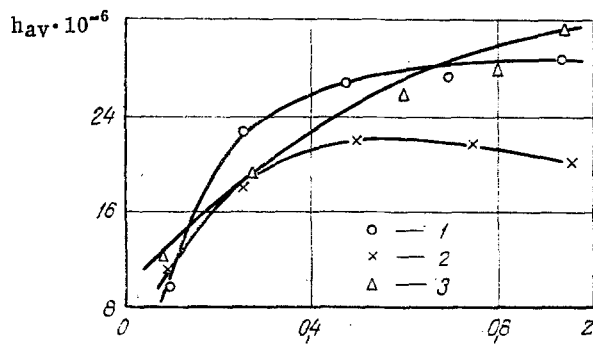


Fig. 3. Distributions of h_{av} along the channel for different EAH: 1) cylindrical channel [5], $I = 180$ A, $G_a = 0.8 \cdot 10^{-3}$ kg·sec⁻¹, $G_o = 0.5 \cdot 10^{-3}$ kg·sec⁻¹, $G_i = 0.06 \cdot 10^{-3}$ kg·sec⁻¹; 2) expanding channel [2], $I = 200$ A, $G_a = 1.85 \cdot 10^{-3}$ kg·sec⁻¹, $G_o = 0.32 \cdot 10^{-3}$ kg·sec⁻¹, $G_i = 0.17 \cdot 10^{-3}$ kg·sec⁻¹; 3) narrowing channel, $I = 200$ A, $G_a = 0.8 \cdot 10^{-3}$ kg·sec⁻¹, $G_o = 0.5 \cdot 10^{-3}$ kg·sec⁻¹, $G_i = 0.06 \cdot 10^{-3}$ kg·sec⁻¹; h_{av} , J·kg⁻¹.

The correctness of the theoretical results was tested experimentally in an EAH where the radius of the arc chamber decreases along the z axis by the law $R = R_0 \sqrt{1 + k_2 \lambda z}$, $k_2 = -6$ m⁻¹ [3]. The air was injected in the initial cross section of the channel and in equal amounts through the gaps between sections of the electrically neutral lining.

In Fig. 1 we present an experimental distribution of the electric field strength along the z axis (curve 1), obtained in [3] with $I = 150$ A, $G_a = 0.8 \cdot 10^{-3}$ kg·sec⁻¹, $G_o = 0.2 \cdot 10^{-3}$ kg·sec⁻¹, and $G_i = 0.12 \cdot 10^{-3}$ kg·sec⁻¹. Theoretical curves of the distribution of electric field strength for the EAH studied are presented in the same figure. Curve 2 was calculated for the case when the radius of the arc does not vary along the channel. The calculation was made by Eq. (7). The average value of k_1 was 75 m⁻¹. As is seen, neglecting the variation of ζ leads to a considerable departure of the calculation from the experimental data. Curve 3 was calculated by Eq. (2) for the region of $z > 0.3$. The law of variation of the radius of the arc was taken in the form $\zeta^2 = \zeta_0^2 (1 + k_1 \lambda z)^{-1}$. The value of k_1 was 22 m⁻¹. In the calculations we took $\zeta_0 = 0.7$, $\lambda = 10.5 \cdot 10^{-2}$ m, $\sigma_S = 0.32$ V⁻², and $h_S = 6.7 \cdot 10^3$ m·sec·kg⁻¹. The departure of curve 3 from that of the experiment does not exceed 10%. Thus, the allowance for the variation of ζ along the EAH axis leads to satisfactory agreement between theory and experiment.

The experiments showed that h_{av} at the EAH exit grows almost linearly as a function of the current. The dependences of h_{av} and η for three values of G_a with $G_i = \text{const}$ are shown in Fig. 2. It is seen from the figure that the average-mass enthalpy decreases slightly with an increase in the gas flow rate. This evidently promotes the passage of a considerable fraction of the gas through the column of the arc. As would be expected, the efficiency of an EAH grows with an increase in G_a .

The distributions of h_{av} along the z axis for different EAH are presented in Fig. 3. A comparison of the distribution curves of the average-mass enthalpy shows that h_{av} reaches the limiting values very rapidly in EAH with cylindrical and expanding channels, while for an EAH with a narrowing channel it grows continuously along the z axis. Thus, profiling of the channel allows one to obtain different distributions of h_{av} , and one can use one or another EAH depending on the concrete conditions.

The theoretical equations obtained in the paper can be used in the design of industrial EAH.

NOTATION

I , current, A; E , electric field strength, V·m⁻¹; η , thermal efficiency of EAH; R , channel radius, m; ζ , radius of arc, normalized to R_0 ; λ , length of channel, m; r, z , cylindrical coordinates, normalized to R_0 and λ , respectively (the z axis is directed along the column in the direction of plasma flow); q , heat flux through a unit length of the arc chamber, W·m⁻¹; G_a , total gas flow rate through EAH, kg·sec⁻¹; G_o , gas flow rate in initial cross section of

EAH arc chamber, $\text{kg}\cdot\text{sec}^{-1}$; G_1 , flow rate of gas injected between sections, $\text{kg}\cdot\text{sec}^{-1}$; G^* , gas flow rate through arc column, $\text{kg}\cdot\text{sec}^{-1}$; σ , electrical conductivity, Ω^{-1} ; h , enthalpy, $\text{J}\cdot\text{kg}^{-1}$; S , heat-conduction function, $\text{W}\cdot\text{m}^{-1}$; h_{av} , average-mass enthalpy, $\text{J}\cdot\text{kg}^{-1}$; h_* , S_* , values of h and S at boundary of arc column, $\text{J}\cdot\text{kg}^{-1}$, $\text{W}\cdot\text{m}^{-1}$; 0 and ∞ pertain to quantities at $z = 0$ and as $z \rightarrow \infty$.

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INFLUENCE OF SCATTERING ANISOTROPY ON THE EMISSION OF TWO-PHASE MEDIA

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An approximate solution of the transfer equation in a plane medium with anisotropic scattering is compared with the exact solution. It is shown that the approximate solution agrees well with the exact solution.

Great attention is now being paid to the problem of allowing for an arbitrary scattering indicatrix in the solution of the equation of radiation transfer in two-phase media [1-5]. The solution of this problem is connected with a large volume of computational work. The development of approximate methods of solving the radiation-transfer equation with allowance for scattering anisotropy naturally becomes very desirable. For example, in [6] approximate solutions are presented for the determination of the hemispherical and directional emissivities of two-phase plane media in which the scattering anisotropy is allowed for by the parameter β :

$$\beta = \beta(\tau) = \frac{\int_{-1}^0 d\mu \int_0^1 p(\mu, \mu') I(\tau, \mu') d\mu'}{2 \int_0^1 I(\tau, \mu) d\mu} = \frac{\int_0^1 d\mu \int_{-1}^0 p(\mu, \mu') I(\tau, \mu') d\mu'}{2 \int_{-1}^0 I(\tau, \mu) d\mu}. \quad (1)$$

Here $I(\tau, \mu)$ is the radiation intensity at the point τ and in the direction $\theta = \arccos \mu$, $\tau = \int_0^z (\kappa + \sigma) dz$ is the optical depth of the layer, κ and σ are the coefficients of absorption and scattering, respectively, while $p(\mu, \mu')$ is the indicatrix for radiation scattering on an elementary volume. In a first approximation the quantity β can be set as constant,

$$\beta \cong \frac{1}{2} \int_{-1}^0 d\mu \int_0^1 p(\mu, \mu') d\mu' = \frac{1}{2} \int_0^1 d\mu \int_{-1}^0 p(\mu, \mu') d\mu'. \quad (2)$$

As is known [6-8], in the investigation of multiple scattering processes one can use a scattering indicatrix in the form

$$p(\mu, \mu') = a + 2(1-a) \delta(\mu - \mu'). \quad (3)$$

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